Mathematical Excursions

FOURTH EDITION

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Mathematical Excursions FOURTH EDITION

AUFMANN · LOCKWOOD · NATION · CLEGG

After teaching liberal arts mathematics classes using traditional texts, we became convinced that a liberal arts mathematics text was needed that included several features designed to increase student success by promoting more active student involvement in the learning process. With this in mind, we have created a text with the features outlined below, each designed to get you actively involved. We encourage you to become familiar with these features so that you can use them to enjoy a quality learning experience and the successful completion of this course.

— RICHARD AUFMANN, JOANNE LOCKWOOD, RICHARD NATION, DANIEL CLEGG

Mathematical Excursions is written in an interactive style that provides you with an opportunity to practice a concept as it is presented.

EXAMPLE 2 **Calculate Simple Interest**

Calculate the simple interest due on a 3-month loan of \$2000 if the interest rate is 6.5%.

Solution

Use the simple interest formula. Substitute the values $P = 2000$ and $r = 6.5\% = 0.065$ into the formula. Because the interest rate is an annual rate, the time must be measured in years: $t = \frac{3 \text{ months}}{1 \text{ year}} = \frac{3 \text{ months}}{12 \text{ months}} = \frac{3}{12}$.

$$I = Prt$$
$$I = 2000(0.065)\left(\frac{3}{12}\right)$$
$$I = 32.5$$

The simple interest due is \$32.50.

CHECK YOUR PROGRESS 2 Calculate the simple interest due on a 4-month loan of \$1500 if the interest rate is 5.25%.

Solution See page S37.

Check your progress 2, *page 621*

$P = 1500,\ r = 5.25\% = 0.0525$

$t = \frac{4 \text{ months}}{1 \text{ year}} = \frac{4 \text{ months}}{12 \text{ months}} = \frac{4}{12}$

$I = Prt$

$I = 1500(0.0525)\left(\frac{4}{12}\right)$

$I = 26.25$

The simple interest due is \$26.25.

Relevant Examples are accompanied by step-by-step solutions.

Check Your Progress allows you to immediately check your understanding of a concept.

A **complete solution** of the Check Your Progress is given in an appendix. This allows you to check your solution.

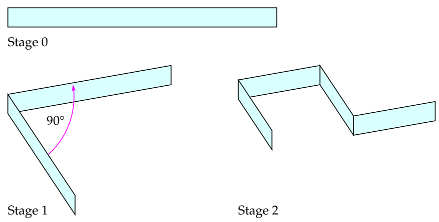
EXCURSION

The Heighway Dragon Fractal

In this Excursion, we illustrate two methods of constructing the stages of a fractal known as the *Heighway dragon*.

The Heighway Dragon via Paper Folding

The first few stages of the Heighway dragon fractal can be constructed by the repeated folding of a strip of paper. In the following discussion, we use a 1-inch-wide strip of paper that is 14 in. in length as stage 0. To create stage 1 of the dragon fractal, just fold the strip in half and open it so that the fold forms a right angle (see Figure 7.29). To create stage 2, fold the original strip twice. The second fold should be in the same direction as the first fold. Open the paper so that each of the folds forms a right angle. Continue the iterative process of making an additional fold in the same direction as the first fold and then forming a right angle at each fold to produce additional stages. See Figure 7.29.



Excursions give you the opportunity to take the concepts from the section and expand on them or apply them in another setting. This promotes a deeper understanding of the concepts in the section.

Many exercises are suitable for **cooperative learning**, providing opportunities to work with others.

19. If a pair of regular dice is tossed once, use the expectation formula to determine the expected sum of the numbers on the upward faces of the 2 dice.

20. Consider rolling a pair of unusual dice, for which the faces have the number of pips indicated.

Die 1: {0, 0, 0, 6, 6, 6}
Die 2: {1, 2, 3, 4, 5, 6}

a. List the sample space for the experiment.
b. Compute the probability of each possible sum of the upward faces on the dice.
c. What is the expected value of the sum of the numbers on the upward faces of the 2 dice?

21. Two dice, one labeled 1, 2, 2, 3, 3, 4 and the other labeled 1, 3, 4, 5, 6, 8, are rolled once. Use the formula for expectation to determine the expected sum of the numbers on the upward faces of the 2 dice. Dice such as these are called *Sicherman dice*.

22. Suppose you purchase a ticket for a prize and your expectation is −$1. What is the meaning of this expectation?

23. Efron's dice Suppose you are offered 1 of 2 pairs of dice, a red pair or a green pair, that are labeled as follows.

Red die 1: 0, 0, 4, 4, 4, 4
Red die 2: 2, 3, 3, 9, 10, 11
Green die 1: 3, 3, 3, 3, 3, 3
Green die 2: 0, 1, 7, 8, 8, 8

After you choose, your friend will receive the other pair. Which pair should you choose if you are going to play a game in which each of you rolls your dice and the player with the higher sum wins? Dice such as these are part of a set of 4 pairs of dice called *Efron's dice*. Which pair should you choose? Explain.

24. Lotteries The PowerBall lottery commission chooses 5 white balls from a drum containing 69 balls marked with the numbers 1 through 69, and 1 red ball from a separate drum containing 26 balls. The following table shows the approximate odds of winning certain prizes if the numbers you choose match those chosen by the lottery commission.

Match	Prize	Odds
○○○○○ + ●	Grand Prize	1 in 292,201,338.00
○○○○○	$1,000,000	1 in 11,688,053.52
○○○○ + ●	$50,000	1 in 913,129.18
○○○○	$100	1 in 36,525.17
○○○ + ●	$100	1 in 14,494.11
○○○	$7	1 in 579.76
○○ + ●	$7	1 in 701.33
○ + ●	$4	1 in 91.98
●	$4	1 in 38.32

The overall odds of winning a prize are 1 in 24.87.
The odds presented here are based on a $2 play
(rounded to two decimal places).

SOURCE: http://www.powerball.com/powerball/pb_prizes.asp

Assuming the jackpot for a certain drawing is $150 million, what is your expectation for the jackpot if you purchase 1 ticket for $2? Round to the nearest cent. Assume the jackpot is not split among multiple winners.

A variety of **End-of-Chapter** features help you prepare for a test.

The **Chapter Summary** reviews the major concepts discussed in the chapter. For each concept, there is a reference to a worked example illustrating how the concept is used and at least one exercise in the Chapter Review Exercises relating to that concept.

CHAPTER 11 SUMMARY

The following table summarizes essential concepts in this chapter. The references given in the right-hand column list Examples and Exercises that can be used to test your understanding of a concept.

11.1 Simple Interest	
Simple Interest Formula The simple interest formula is $I = Prt$, where I is the interest, P is the principal, r is the interest rate, and t is the time period.	See **Examples 2, 4, and 5** on pages 621 and 622, and then try Exercises 1, 2, and 3 on page 683.

Chapter Review Exercises help you review all of the concepts in the chapter. Answers to all the Chapter Review Exercises are in the answer section, along with a reference to the section from which the exercise was taken. If you miss an exercise, use that reference to review the concept.

CHAPTER 11 REVIEW EXERCISES

1. Simple Interest Calculate the simple interest due on a 4-month loan of $2750 if the interest rate is 6.75%.

2. Simple Interest Find the simple interest due on an 8-month loan of $8500 if the interest rate is 1.15% per month.

3. Simple Interest What is the simple interest earned in 120 days on a deposit of $4000 if the interest rate is 6.75%?

4. Maturity Va
simple interes
rate is 10.4%.

5. Simple Inte
on a 3-month
interest rate.

6. Compound Amount Calculate the compound amount when $3000 is deposited in an account earning 6.6% interest, compounded monthly, for 3 years.

7. Compound Amount What is the compound amount when $6400 is deposited in an account earning an interest rate of 6%, compounded quarterly, for 10 years?

8. Future Value Find the future value of $6000 earning 9% interest, compounded daily, for 3 years.

CHAPTER 11 REVIEW EXERCISES *page 683*

1. $61.88 [Sec. 11.1] **2.** $782 [Sec. 11.1] **3.** $90 [Sec. 11.1] **4.** $7218.40 [Sec. 11.1] **5.** 7.5% [Sec. 11.1]
6. $3654.90 [Sec. 11.2] **7.** $11,609.72 [Sec. 11.2] **8.** $7859.52 [Sec. 11.2] **9.** $200.23 [Sec. 11.2]
10. $10,683.29 [Sec. 11.2] **11. a.** $11,318.23 **b.** $3318.23 [Sec. 11.2] **12.** $19,225.50 [Sec. 11.2]
13. 1.1% [Sec. 11.4] **14.** $9000 [Sec. 11.4] **15.** $1.59 [Sec. 11.2] **16.** $43,650.68 [Sec. 11.2]
17. 6.06% [Sec. 11.2] **18.** 5.4% compounded semiannually [Sec. 11.2] **19.** $431.16 [Sec. 11.3]
20. $6.12 [Sec. 11.3] **21. a.** $259.38 **b.** 12.75% [Sec. 11.3] **22. a.** $36.03 **b.** 12.9% [Sec. 11.3]
23. $45.41 [Sec. 11.3] **24. a.** $10,092.69 **b.** $2018.54 **c.** $253.01 [Sec. 11.3] **25.** $664.40 [Sec. 11.3]
26. a. $540.02 **b.** $12,196.80 [Sec. 11.3] **27. a.** Profit of $5325 **b.** $256.10 [Sec. 11.4]
28. 200 shares [Sec. 11.4] **29.** $99,041 [Sec. 11.5] **30. a.** $1659.11 **b.** $597,279.60 **c.** $341,479.60 [Sec. 11.5]
31. a. $1396.69 **b.** $150,665.74 [Sec. 11.5] **32.** $2658.53 [Sec. 11.5] **33.** $288.62 [Sec. 11.3]

The **Chapter Test** gives you a chance to practice a possible test for the chapter. Answers to all Chapter Test questions are in the answer section, along with a section reference for the question.

CHAPTER 11 TEST

1. Simple Interest Calculate the simple interest due on a 3-month loan of $5250 if the interest rate is 8.25%.

2. Simple Interest Find the simple interest earned in 180 days on a deposit of $6000 if the interest rate is 6.75%.

3. Maturity Value Calculate the maturity value of a simple interest, 200-day loan of $8000 if the interest rate is 9.2%.

4. Simple Interest Rate The simple interest charged

10. Bonds Suppose you purchase a $5000 bond that has a 3.8% coupon and a 10-year maturity. Calculate the total of the interest payments that you will receive.

11. Inflation In 2016, the median value of a single-family house was $224,000. Use an annual inflation rate of 4.3% to calculate the median value of a single family house in 2029. (*Source:* money.cnn.com)

12. Effective Interest Rate Calculate the effective interest rate of 6.25% compounded quarterly. Round t
the nearest hundredth of a percent.

For the Chapter Test, besides a reference to the section from which an exercise was taken, there is a reference to an example that is similar to the exercise.

CHAPTER 11 TEST *page 685*

1. $108.28 [Sec. 11.1, Example 2] **2.** $202.50 [Sec. 11.1, Example 1] **3.** $8408.89 [Sec. 11.1, Example 6]
4. 9% [Sec. 11.1, Example 5] **5.** $7340.87 [Sec. 11.2, Check Your Progress 2] **6.** $312.03 [Sec. 11.2, Example 4]
7. a. $15,331.03 **b.** $4831.03 [Sec. 11.1, Example 6] **8.** $21,949.06 [Sec. 11.2, Example 6] **9.** 1.2% [Sec. 11.4, Example 2]
10. $1900 [Sec. 11.4, Example 4] **11.** $387,207.74 [Sec. 11.2, Check Your Progress 8] **12.** 6.40% [Sec. 11.2, Check Your Progress 10]
13. 4.6% compounded semiannually [Sec. 11.2, Example 11] **14.** $7.79 [Sec. 11.3, Example 1] **15. a.** $48.56
b. 16.6% [Sec. 11.3, Example 2] **16.** $56.49 [Sec. 11.3, Example 3] **17. a.** Loss of $4896 **b.** $226.16 [Sec. 11.4, Example 3]
18. 208 shares [Sec. 11.4, Example 5] **19. a.** $6985.94 **b.** $1397.19 **c.** $174.62 [Sec. 11.3, Example 4]
20. $60,083.50 [Sec. 11.5, Example 1] **21. a.** $1530.69 [Sec. 11.5, Example 2a] **b.** $221,546.46 [Sec. 11.5, Example 4]
22. $2595.97 [Sec. 11.5, Example 2a, Example 5]

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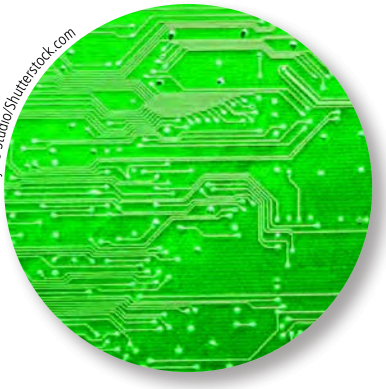
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Preface

Mathematical Excursions is about mathematics as a system of knowing or understanding our surroundings. It is similar to an English literature textbook, an introduction to philosophy textbook, or perhaps an introductory psychology textbook. Each of those books provides glimpses into the thoughts and perceptions of some of the world's greatest writers, philosophers, and psychologists. Reading and studying their thoughts enables us to better understand the world we inhabit.

In a similar way, *Mathematical Excursions* provides glimpses into the nature of mathematics and how it is used to understand our world. This understanding, in conjunction with other disciplines, contributes to a more complete portrait of the world. Our contention is that:

- Planning a shopping trip to several local stores, or several cities scattered across Europe, is more interesting when one has knowledge of efficient routes, which is a concept from the field of graph theory.
- Problem solving is more enjoyable after you have studied a variety of problem-solving techniques and have practiced using George Polya's four-step, problem-solving strategy.
- The challenges of sending information across the Internet are better understood by examining prime numbers.
- The perils of radioactive waste take on new meaning with knowledge of exponential functions.
- Generally, knowledge of mathematics strengthens the way we know, perceive, and understand our surroundings.

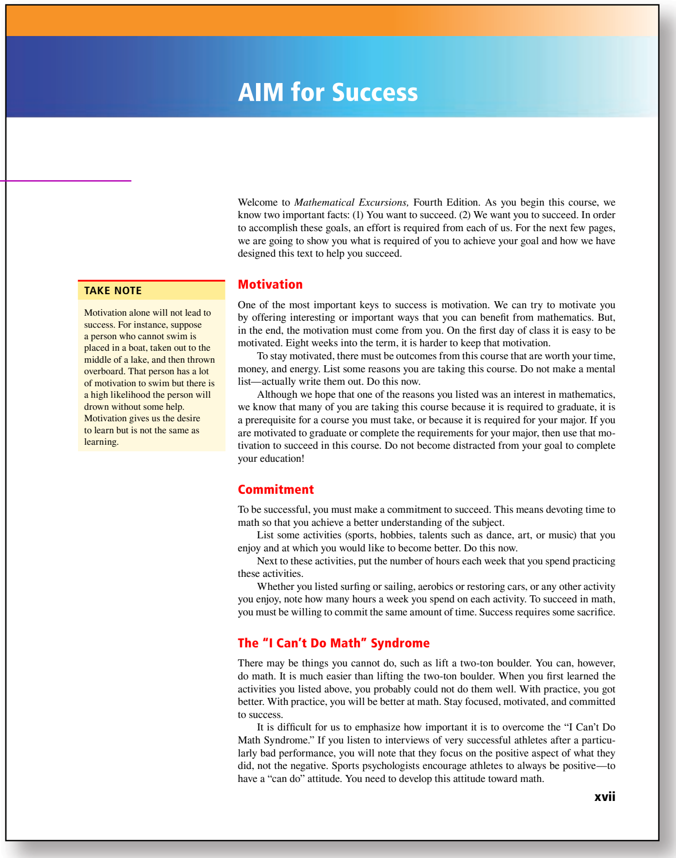
The central purpose of *Mathematical Excursions* is to explore those facets of mathematics that will strengthen your quantitative understandings of our environs. We hope you enjoy the journey.

Updates to This Edition

- Application Examples, Exercises, and Excursions have been updated to reflect recent data and trends.
- Expanded Chapter 7 with the addition of a section on measurement.
- Extension exercises have been consolidated and streamlined.

Interactive Method

The AIM FOR SUCCESS STUDENT PREFACE explains what is required of a student to be successful and how this text has been designed to foster student success. This "how to use this text" preface can be used as a lesson on the first day of class or as a project for students to complete to strengthen their study skills.



AIM for Success

Welcome to *Mathematical Excursions,* Fourth Edition. As you begin this course, we know two important facts: (1) You want to succeed. (2) We want you to succeed. In order to accomplish these goals, an effort is required from each of us. For the next few pages, we are going to show you what is required of you to achieve your goal and how we have designed this text to help you succeed.

TAKE NOTE

Motivation alone will not lead to success. For instance, suppose a person who cannot swim is placed in a boat, taken out to the middle of a lake, and then thrown overboard. That person has a lot of motivation to swim but there is a high likelihood the person will drown without some help. Motivation gives us the desire to learn but is not the same as learning.

Motivation

One of the most important keys to success is motivation. We can try to motivate you by offering interesting or important ways that you can benefit from mathematics. But, in the end, the motivation must come from you. On the first day of class it is easy to be motivated. Eight weeks into the term, it is harder to keep that motivation.

To stay motivated, there must be outcomes from this course that are worth your time, money, and energy. List some reasons you are taking this course. Do not make a mental list—actually write them out. Do this now.

Although we hope that one of the reasons you listed was an interest in mathematics, we know that many of you are taking this course because it is required to graduate, it is a prerequisite for a course you must take, or because it is required for your major. If you are motivated to graduate or complete the requirements for your major, then use that motivation to succeed in this course. Do not become distracted from your goal to complete your education!

Commitment

To be successful, you must make a commitment to succeed. This means devoting time to math so that you achieve a better understanding of the subject.

List some activities (sports, hobbies, talents such as dance, art, or music) that you enjoy and at which you would like to become better. Do this now.

Next to these activities, put the number of hours each week that you spend practicing these activities.

Whether you listed surfing or sailing, aerobics or restoring cars, or any other activity you enjoy, note how many hours a week you spend on each activity. To succeed in math, you must be willing to commit the same amount of time. Success requires some sacrifice.

The "I Can't Do Math" Syndrome

There may be things you cannot do, such as lift a two-ton boulder. You can, however, do math. It is much easier than lifting the two-ton boulder. When you first learned the activities you listed above, you probably could not do them well. With practice, you got better. With practice, you will be better at math. Stay focused, motivated, and committed to success.

It is difficult for us to emphasize how important it is to overcome the "I Can't Do Math Syndrome." If you listen to interviews of very successful athletes after a particularly bad performance, you will note that they focus on the positive aspect of what they did, not the negative. Sports psychologists encourage athletes to always be positive—to have a "can do" attitude. You need to develop this attitude toward math.

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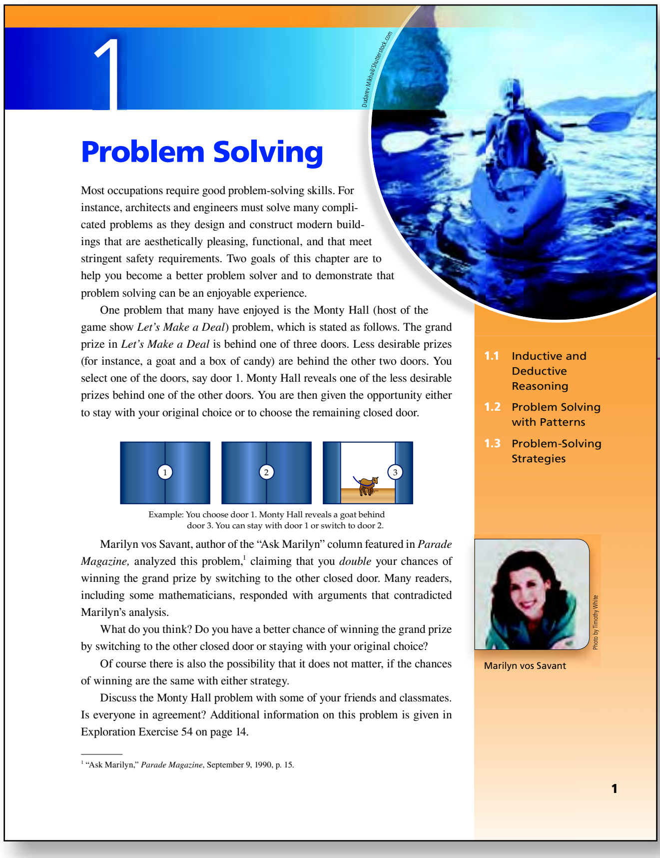

1

Problem Solving

Most occupations require good problem-solving skills. For instance, architects and engineers must solve many complicated problems as they design and construct modern buildings that are aesthetically pleasing, functional, and that meet stringent safety requirements. Two goals of this chapter are to help you become a better problem solver and to demonstrate that problem solving can be an enjoyable experience.

One problem that many have enjoyed is the Monty Hall (host of the game show *Let's Make a Deal*) problem, which is stated as follows. The grand prize in *Let's Make a Deal* is behind one of three doors. Less desirable prizes (for instance, a goat and a box of candy) are behind the other two doors. You select one of the doors, say door 1. Monty Hall reveals one of the less desirable prizes behind one of the other doors. You are then given the opportunity either to stay with your original choice or to choose the remaining closed door.

Example: You choose door 1. Monty Hall reveals a goat behind door 3. You can stay with door 1 or switch to door 2.

Marilyn vos Savant, author of the "Ask Marilyn" column featured in *Parade Magazine,* analyzed this problem,[1] claiming that you *double* your chances of winning the grand prize by switching to the other closed door. Many readers, including some mathematicians, responded with arguments that contradicted Marilyn's analysis.

What do you think? Do you have a better chance of winning the grand prize by switching to the other closed door or staying with your original choice?

Of course there is also the possibility that it does not matter, if the chances of winning are the same with either strategy.

Discuss the Monty Hall problem with some of your friends and classmates. Is everyone in agreement? Additional information on this problem is given in Exploration Exercise 54 on page 14.

[1] "Ask Marilyn," *Parade Magazine*, September 9, 1990, p. 15.

Marilyn vos Savant

1

Each CHAPTER OPENER includes a list of sections that can be found within the chapter and includes an anecdote, description, or explanation that introduces the student to a topic in the chapter.

EXAMPLE 3 **Write Compound Statements in Symbolic Form**

Consider the following simple statements.

p: Today is Friday.
q: It is raining.
r: I am going to a movie.
s: I am not going to the basketball game.

Write the following compound statements in symbolic form.

a. Today is Friday and it is raining.
b. It is not raining and I am going to a movie.
c. I am going to the basketball game or I am going to a movie.
d. If it is raining, then I am not going to the basketball game.

Solution

a. $p \wedge q$ **b.** $\sim q \wedge r$ **c.** $\sim s \vee r$ **d.** $q \rightarrow s$

CHECK YOUR PROGRESS 3 Use p, q, r, and s as defined in Example 3 to write the following compound statements in symbolic form.

a. Today is not Friday and I am going to a movie.
b. I am going to the basketball game and I am not going to a movie.
c. I am going to the movie if and only if it is raining.
d. If today is Friday, then I am not going to a movie.

Solution See page S8.

In the next example, we translate symbolic statements into English sentences.

EXAMPLE 4 **Translate Symbolic Statements**

Consider the following statements.

p: The game will be played in Atlanta.
q: The game will be shown on CBS.
r: The game will not be shown on ESPN.
s: The Mets are favored to win.

Write each of the following symbolic statements in words.

a. $q \wedge p$ **b.** $\sim r \wedge s$ **c.** $s \leftrightarrow \sim p$

Solution

a. The game will be shown on CBS and the game will be played in Atlanta.
b. The game will be shown on ESPN and the Mets are favored to win.
c. The Mets are favored to win if and only if the game will not be played in Atlanta.

CHECK YOUR PROGRESS 4 Consider the following statements.

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e: All men are created equal.
t: I am trading places.
a: I get Abe's place.
g: I get George's place.

Use the above information to translate the dialogue in the speech bubbles at the left.

Solution See page S8.

Each section contains a variety of WORKED EXAMPLES. Each example is given a title so that the student can see at a glance the type of problem that is being solved. Most examples include annotations that assist the student in moving from step to step, and the final answer is in color in order to be readily identifiable.

Following each worked example is a CHECK YOUR PROGRESS exercise for the student to work. By solving this exercise, the student actively practices concepts as they are presented in the text. For each Check Your Progress exercise, there is a detailed solution in the Solutions appendix.

At various places throughout the text, a QUESTION is posed about the topic that is being discussed. This question encourages students to pause, think about the current discussion, and answer the question. Students can immediately check their understanding by referring to the ANSWER to the question provided in a footnote on the same page. This feature creates another opportunity for the student to interact with the textbook.

We can find any term after the second term of the Fibonacci sequence by computing the sum of the previous two terms. However, this procedure of adding the previous two terms can be tedious. For instance, what is the 100th term or the 1000th term of the Fibonacci sequence? To find the 100th term, we need to know the 98th and 99th terms. To find the 1000th term, we need to know the 998th and 999th terms. Many mathematicians tried to find a nonrecursive nth-term formula for the Fibonacci sequence without success, until a formula was discovered by Jacques Binet in 1843. Binet's formula is given in Exercise 23 of this section.

QUESTION What happens if you try to use a difference table to determine Fibonacci numbers?

EXAMPLE 4 **Determine Properties of Fibonacci Numbers**

Determine whether each of the following statements about Fibonacci numbers is true or false. *Note:* The first 10 terms of the Fibonacci sequence are 1, 1, 2, 3, 5, 8, 13, 21, 34, and 55.

a. If n is even, then F_n is an odd number. **b.** $2F_n - F_{n-2} = F_{n+1}$ for $n \geq 3$

ANSWER The difference table for the numbers in the Fibonacci sequence does not contain a row of differences that are all the same constant.

Each section ends with an EXCURSION along with corresponding EXCURSION EXERCISES. These activities engage students in the mathematics of the section. Some Excursions are designed as in-class cooperative learning activities that lend themselves to a hands-on approach. They can also be assigned as projects or extra credit assignments. The Excursions are a unique and important feature of this text. They provide opportunities for students to take an active role in the learning process.

EXCURSION

Earned Run Average

One measure of a pitcher's success is earned run average. **Earned run average (ERA)** is the number of earned runs a pitcher gives up for every nine innings pitched. The definition of an earned run is somewhat complicated, but basically an earned run is a run that is scored as a result of hits and base running that involves no errors on the part of the pitcher's team. If the opposing team scores a run on an error (for example, a fly ball that should have been caught in the outfield was fumbled), then that run is not an earned run.

A proportion is used to calculate a pitcher's ERA. Remember that the statistic involves the number of earned runs per *nine innings*. The answer is always rounded to the nearest hundredth. Here is an example.

During the 2015 baseball season, Clayton Kershaw gave up 55 earned runs and pitched 232.2 innings for the Los Angeles Dodgers. To calculate Clayton Kershaw's ERA, let $x =$ the number of earned runs for every nine innings pitched. Write a proportion and then solve it for x.

$$\frac{55 \text{ earned runs}}{232.2 \text{ innings}} = \frac{x}{9 \text{ innings}}$$
$$55 \cdot 9 = 232.2 \cdot x$$
$$495 = 232.2x$$
$$\frac{495}{232.2} = \frac{232.2x}{232.2}$$
$$2.13 \approx x$$

Clayton Kershaw's ERA for the 2015 season was 2.13.

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Clayton Kershaw

Earned Run Average Leaders		
Major League Baseball		
Year	Player, club	ERA
2005	Roger Clemens, Houston	1.87
2006	Johan Santana, Minnesota	2.77
2007	Jake Peavy, San Diego	2.54
2008	Johan Santana, New York	2.53
2009	Zack Greinke, Kansas City	2.16
2010	Felix Hernandez, Seattle	2.27
2011	Clayton Kershaw, Los Angeles	2.28
2012	Clayton Kershaw, Los Angeles	2.53
2013	Clayton Kershaw, Los Angeles	1.83
2014	Clayton Kershaw, Los Angeles	1.77
2015	Zack Greinke, Los Angeles	1.66

EXCURSION EXERCISES

1. In 1979, his rookie year, Jeff Reardon pitched 21 innings for the New York Mets and gave up four earned runs. Calculate Reardon's ERA for 1979.
2. Roger Clemens's first year with the Boston Red Sox was 1984. During that season, he pitched 133.1 innings and gave up 64 earned runs. Calculate Clemens's ERA for 1984.
3. In 1987, Nolan Ryan had the lowest ERA of any pitcher in the major leagues. He gave up 65 earned runs and pitched 211.2 innings for the Houston Astros. Calculate Ryan's ERA for 1987.
4. During the 2015 season, Jake Arrieta of the Baltimore Orioles pitched 229 innings and had an ERA of 1.77. How many earned runs did he give up during the season?
5. Find the necessary statistics for a pitcher on your "home team," and calculate that pitcher's ERA.

EXERCISE SET 4.3

■ In the following exercises that involve weighted voting systems for voters A, B, C, ..., the systems are given in the form $\{q: w_1, w_2, w_3, w_4, \ldots, w_n\}$. The weight of voter A is w_1, the weight of voter B is w_2, the weight of voter C is w_3, and so on.

1. A weighted voting system is given by {6: 4, 3, 2, 1}.
   a. What is the quota?
   b. How many voters are in this system?
   c. What is the weight of voter B?
   d. What is the weight of the coalition {A, C}?
   e. Is {A, D} a winning coalition?
   f. Which voters are critical voters in the coalition {A, C, D}?
   g. How many coalitions can be formed?
   h. How many coalitions consist of exactly two voters?
2. A weighted voting system is given by {16: 8, 7, 4, 2, 1}.
   a. What is the quota?
   b. How many voters are in this system?
   c. What is the weight of voter C?
   d. What is the weight of the coalition {B, C}?
   e. Is {B, C, D, E} a winning coalition?
   f. Which voters are critical voters in the coalition {A, B, D}?
   g. How many coalitions can be formed?
   h. How many coalitions consist of exactly three voters?

■ In Exercises 3 to 12, calculate, if possible, the Banzhaf power index for each voter. Round to the nearest hundredth.

3. {6: 4, 3, 2}
4. {10: 7, 6, 4}
5. {10: 7, 3, 2, 1}
6. {14: 7, 5, 1, 1}
7. {19: 14, 12, 4, 3, 1}
8. {3: 1, 1, 1, 1}
9. {18: 18, 7, 3, 3, 1, 1}
10. {14: 6, 6, 4, 3, 1}
11. {80: 50, 40, 30, 25, 5}
12. {85: 55, 40, 25, 5}
13. Which, if any, of the voting systems in Exercises 3 to 12 is
    a. a dictatorship?
    b. a veto power system? *Note:* A voting system is a veto power system if any of the voters has veto power.
    c. a null system?
    d. a one-person, one-vote system?
14. Explain why it is impossible to calculate the Banzhaf power index for any voter in the null system {8: 3, 2, 1, 1}.
15. **Music Education** A music department consists of a band director and a music teacher. Decisions on motions are made by voting. If both members vote in favor of a motion, it passes. If both members vote against a motion, it fails. In the event of a tie vote, the principal of the school votes to break the tie. For this voting scheme, determine the Banzhaf power index for each department member and for the principal. *Hint:* See Example 3, page 214.
16. Four voters, A, B, C, and D, make decisions by using the voting scheme {4: 3, 1, 1, 1}, except when there is a tie. In the event of a tie, a fifth voter, E, casts a vote to break the tie. For this voting scheme, determine the Banzhaf power index for each voter, including voter E. *Hint:* See Example 3, page 214.
17. **Criminal Justice** In a criminal trial, each of the 12 jurors has one vote and all of the jurors must agree to reach a verdict. Otherwise the judge will declare a mistrial.
    a. Write the weighted voting system, in the form $\{q: w_1, w_2, w_3, w_4, \ldots, w_{12}\}$, used by these jurors.
    b. Is this weighted voting system a one-person, one-vote system?
    c. Is this weighted voting system a veto power system?
    d. Explain an easy way to determine the Banzhaf power index for each voter.
18. **Criminal Justice** In California civil court cases, each of the 12 jurors has one vote and at least 9 of the jury members must agree on the verdict.

    a. Write the weighted voting system, in the form $\{q: w_1, w_2, w_3, w_4, \ldots, w_{12}\}$, used by these jurors.
    b. Is this weighted voting system a one-person, one-vote system?
    c. Is this weighted voting system a veto power system?
    d. Explain an easy way to determine the Banzhaf power index for each voter.

The EXERCISE SETS were carefully written to provide a wide variety of exercises that range from drill and practice to interesting challenges. Exercise sets emphasize skill building, skill maintenance, concepts, and applications. Icons are used to identify various types of exercises.

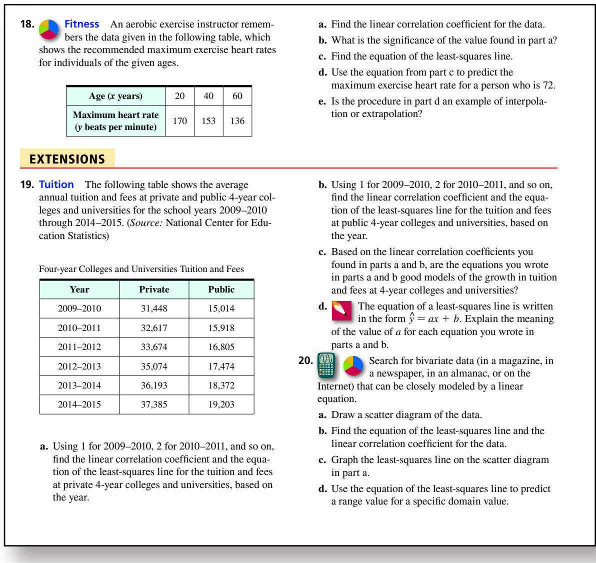
Writing exercises

Data analysis exercises

Graphing calculator exercises

Exercises that require the Internet

EXTENSIONS EXERCISES are placed at the end of each exercise set. These exercises are designed to extend concepts. In most cases these exercises are more challenging and require more time and effort than the preceding exercises.



18. **Fitness** An aerobic exercise instructor remembers the data given in the following table, which shows the recommended maximum exercise heart rates for individuals of the given ages.

Age (*x* years)	20	40	60
Maximum heart rate (*y* beats per minute)	170	153	136

a. Find the linear correlation coefficient for the data.
b. What is the significance of the value found in part a?
c. Find the equation of the least-squares line.
d. Use the equation from part c to predict the maximum exercise heart rate for a person who is 72.
e. Is the procedure in part d an example of interpolation or extrapolation?

EXTENSIONS

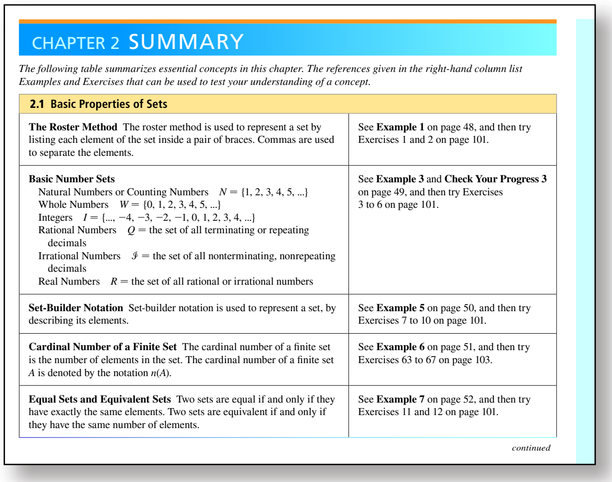
19. **Tuition** The following table shows the average annual tuition and fees at private and public 4-year colleges and universities for the school years 2009–2010 through 2014–2015. (*Source:* National Center for Education Statistics)

Four-year Colleges and Universities Tuition and Fees

Year	Private	Public
2009–2010	31,448	15,014
2010–2011	32,617	15,918
2011–2012	33,674	16,805
2012–2013	35,074	17,474
2013–2014	36,193	18,372
2014–2015	37,385	19,203

a. Using 1 for 2009–2010, 2 for 2010–2011, and so on, find the linear correlation coefficient and the equation of the least-squares line for the tuition and fees at private 4-year colleges and universities, based on the year.
b. Using 1 for 2009–2010, 2 for 2010–2011, and so on, find the linear correlation coefficient and the equation of the least-squares line for the tuition and fees at public 4-year colleges and universities, based on the year.
c. Based on the linear correlation coefficients you found in parts a and b, are the equations you wrote in parts a and b good models of the growth in tuition and fees at 4-year colleges and universities?
d. The equation of a least-squares line is written in the form $\hat{y} = ax + b$. Explain the meaning of the value of a for each equation you wrote in parts a and b.

20. Search for bivariate data (in a magazine, in a newspaper, in an almanac, or on the Internet) that can be closely modeled by a linear equation.
a. Draw a scatter diagram of the data.
b. Find the equation of the least-squares line and the linear correlation coefficient for the data.
c. Graph the least-squares line on the scatter diagram in part a.
d. Use the equation of the least-squares line to predict a range value for a specific domain value.



CHAPTER 2 SUMMARY

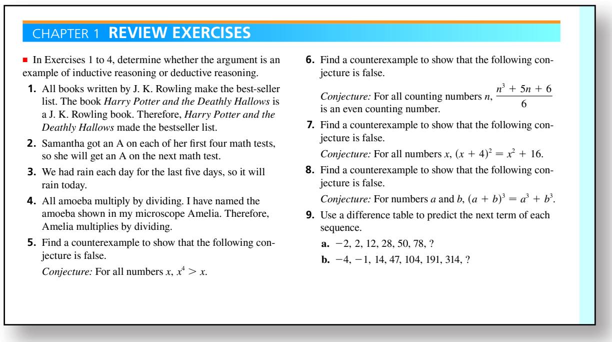
The following table summarizes essential concepts in this chapter. The references given in the right-hand column list Examples and Exercises that can be used to test your understanding of a concept.

2.1 Basic Properties of Sets	
The Roster Method The roster method is used to represent a set by listing each element of the set inside a pair of braces. Commas are used to separate the elements.	See **Example 1** on page 48, and then try Exercises 1 and 2 on page 101.
Basic Number Sets Natural Numbers or Counting Numbers $N = \{1, 2, 3, 4, 5, \ldots\}$ Whole Numbers $W = \{0, 1, 2, 3, 4, 5, \ldots\}$ Integers $I = \{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$ Rational Numbers Q = the set of all terminating or repeating decimals Irrational Numbers $\mathcal{I}$ = the set of all nonterminating, nonrepeating decimals Real Numbers R = the set of all rational or irrational numbers	See **Example 3** and **Check Your Progress 3** on page 49, and then try Exercises 3 to 6 on page 101.
Set-Builder Notation Set-builder notation is used to represent a set, by describing its elements.	See **Example 5** on page 50, and then try Exercises 7 to 10 on page 101.
Cardinal Number of a Finite Set The cardinal number of a finite set is the number of elements in the set. The cardinal number of a finite set A is denoted by the notation $n(A)$.	See **Example 6** on page 51, and then try Exercises 63 to 67 on page 103.
Equal Sets and Equivalent Sets Two sets are equal if and only if they have exactly the same elements. Two sets are equivalent if and only if they have the same number of elements.	See **Example 7** on page 52, and then try Exercises 11 and 12 on page 101.

continued

At the end of each chapter is a CHAPTER SUMMARY that describes the concepts presented in each section of the chapter. Each concept is paired with page numbers of examples that illustrate the concept and exercises that students can use to test their understanding of a concept.

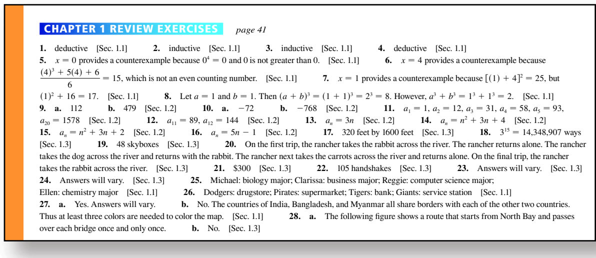
CHAPTER REVIEW EXERCISES are found near the end of each chapter. These exercises were selected to help the student integrate the major topics presented in the chapter. The answers to all the Chapter Review exercises appear in the answer section along with a section reference for each exercise. These section references indicate the section or sections where a student can locate the concepts needed to solve the exercise.



CHAPTER 1 REVIEW EXERCISES

In Exercises 1 to 4, determine whether the argument is an example of inductive reasoning or deductive reasoning.

1. All books written by J. K. Rowling make the best-seller list. The book *Harry Potter and the Deathly Hallows* is a J. K. Rowling book. Therefore, *Harry Potter and the Deathly Hallows* made the bestseller list.
2. Samantha got an A on each of her first four math tests, so she will get an A on the next math test.
3. We had rain each day for the last five days, so it will rain today.
4. All amoeba multiply by dividing. I have named the amoeba shown in my microscope Amelia. Therefore, Amelia multiplies by dividing.
5. Find a counterexample to show that the following conjecture is false.
Conjecture: For all numbers x, $x^4 > x$.
6. Find a counterexample to show that the following conjecture is false.
Conjecture: For all counting numbers n, $\frac{n^3 + 5n + 6}{6}$ is an even counting number.
7. Find a counterexample to show that the following conjecture is false.
Conjecture: For all numbers x, $(x + 4)^2 = x^2 + 16$.
8. Find a counterexample to show that the following conjecture is false.
Conjecture: For numbers a and b, $(a + b)^3 = a^3 + b^3$.
9. Use a difference table to predict the next term of each sequence.
a. −2, 2, 12, 28, 50, 78, ?
b. −4, −1, 14, 47, 104, 191, 314, ?



CHAPTER 1 REVIEW EXERCISES *page 41*

1. deductive [Sec. 1.1] **2.** inductive [Sec. 1.1] **3.** inductive [Sec. 1.1] **4.** deductive [Sec. 1.1] **5.** $x = 0$ provides a counterexample because $0^4 = 0$ and 0 is not greater than 0. [Sec. 1.1] **6.** $x = 4$ provides a counterexample because $\frac{(4)^3 + 5(4) + 6}{6} = 15$, which is not an even counting number. [Sec. 1.1] **7.** $x = 1$ provides a counterexample because $[(1) + 4]^2 = 25$, but $(1)^2 + 16 = 17$. [Sec. 1.1] **8.** Let $a = 1$ and $b = 1$. Then $(a + b)^3 = (1 + 1)^3 = 2^3 = 8$. However, $a^3 + b^3 = 1^3 + 1^3 = 2$. [Sec. 1.1] **9. a.** 112 **b.** 479 [Sec. 1.2] **10. a.** −72 **b.** −768 [Sec. 1.2] **11.** $a_1 = 1$, $a_2 = 12$, $a_3 = 31$, $a_4 = 58$, $a_5 = 93$, $a_{20} = 1578$ [Sec. 1.2] **12.** $a_{11} = 89$, $a_{12} = 144$ [Sec. 1.2] **13.** $a_n = 3n$ [Sec. 1.2] **14.** $a_n = n^2 + 3n + 4$ [Sec. 1.2] **15.** $a_n = n^2 + 3n + 2$ [Sec. 1.2] **16.** $a_n = 5n - 1$ [Sec. 1.2] **17.** 320 feet by 1600 feet [Sec. 1.3] **18.** $3^{15} = 14{,}348{,}907$ ways [Sec. 1.3] **19.** 48 skyboxes [Sec. 1.3] **20.** On the first trip, the rancher takes the rabbit across the river. The rancher returns alone. The rancher takes the dog across the river and returns with the rabbit. The rancher next takes the carrots across the river and returns alone. On the final trip, the rancher takes the rabbit across the river. [Sec. 1.3] **21.** \$300 [Sec. 1.3] **22.** 105 handshakes [Sec. 1.3] **23.** Answers will vary. [Sec. 1.3] **24.** Answers will vary. [Sec. 1.3] **25.** Michael: biology major; Clarissa: business major; Reggie: computer science major; Ellen: chemistry major [Sec. 1.1] **26.** Dodgers: drugstore; Pirates: supermarket; Tigers: bank; Giants: service station [Sec. 1.1] **27. a.** Yes. Answers will vary. **b.** No. The countries of India, Bangladesh, and Myanmar all share borders with each of the other two countries. Thus at least three colors are needed to color the map. [Sec. 1.1] **28. a.** The following figure shows a route that starts from North Bay and passes over each bridge once and only once. **b.** No. [Sec. 1.3]

The CHAPTER TEST exercises are designed to emulate a possible test of the material in the chapter. The answers to all the Chapter Test exercises appear in the answer section along with a section reference and an example reference for each exercise. The section references indicate the section or sections where a student can locate the concepts needed to solve the exercise, and the example references allow students to readily find an example that is similar to a given test exercise.

CHAPTER 3 TEST

1. Determine whether each sentence is a statement.
   - a. Look for the cat.
   - b. Clark Kent is afraid of the dark.
2. Write the negation of each statement. Start each negation with "Some," "No," or "All."
   - a. Some trees are not green.
   - b. No apartments are available.
3. Determine whether each statement is true or false.
   - a. $5 \leq 4$
   - b. $-2 \geq -2$
4. Determine the truth value of each statement given that p is true, q is false, and r is true.
   - a. $(p \vee \sim q) \wedge (\sim r \wedge q)$
   - b. $(r \vee \sim p) \vee [(p \vee \sim q) \leftrightarrow (q \rightarrow r)]$

■ In Exercises 5 and 6, construct a truth table for the given statement.

5. $\sim(p \wedge \sim q) \vee (q \rightarrow p)$ 6. $(r \leftrightarrow \sim q) \wedge (p \rightarrow q)$
7. Use one of De Morgan's laws to write the following in an equivalent form.

   Elle did not eat breakfast and she did not take a lunch break.
8. What is a tautology?
9. Write $p \rightarrow q$ in its equivalent disjunctive form.
10. Determine whether the given statement is true or false. Assume that x, y, and z are real numbers.
    - a. $x = y$ if $|x| = |y|$. b. If $x > y$, then $xz > yz$.

CHAPTER 3 TEST *page 166*

1. a. Not a statement **b.** Statement [Sec. 3.1, Example 1] **2. a.** All trees are green. **b.** Some apartments are available. [Sec. 3.1, Example 2] **3. a.** False **b.** True [Sec. 3.1, Example 6] **4. a.** False **b.** True [Sec. 3.3, Example 3]

p	q	5. [Sec. 3.3, Example 3]
T	T	T
T	F	T
F	T	T
F	F	T

p	q	r	6. [Sec. 3.3, Example 3]
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	F

7. It is not true that Elle ate breakfast or took a lunch break. [Sec. 3.2, Example 5]

Other Key Features

MATH MATTERS The Barber's Paradox


PictureQuest/Corbis Images

Some problems that concern sets have led to paradoxes. For instance, in 1902, the mathematician Bertrand Russell developed the following paradox. "Is the set A of all sets that are not elements of themselves an element of itself?" Both the assumption that A is an element of A and the assumption that A is not an element of A lead to a contradiction. Russell's paradox has been popularized as follows.

> The town barber shaves all males who do not shave themselves, and he shaves only those males. The town barber is a male who shaves. Who shaves the barber?

The assumption that the barber shaves himself leads to a contradiction, and the assumption that the barber does not shave himself also leads to a contradiction.

Math Matters

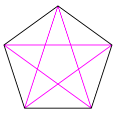
This feature of the text typically contains an interesting sidelight about mathematics, its history, or its applications.

HISTORICAL NOTE


Hutton Archive/Handout/Getty Images

Pythagoras (c. 580 B.C.–520 B.C.) The ancient Greek philosopher and mathematician Pythagoras (pĭ-thăg′ər-əs) formed a secret brotherhood that investigated topics in music, astronomy, philosophy, and mathematics. The Pythagoreans believed that the nature of the universe was directly related to mathematics and that whole numbers and the ratios formed by whole numbers could be used to describe and represent all natural events.

The Pythagoreans were particularly intrigued by the number 5 and the shape of a pentagon. They used the following figure, which is a five-pointed star inside a regular pentagon, as a secret symbol that could be used to identify other members of the brotherhood.



Historical Note

These margin notes provide historical background information related to the concept under discussion or vignettes of individuals who were responsible for major advancements in their fields of expertise.

POINT OF INTEREST


M.C. Escher

Waterfall by M.C. Escher

M.C. Escher (1898–1972) created many works of art that defy logic. In this lithograph, the water completes a full cycle even though the water is always traveling downward.

Point of Interest

These short margin notes provide interesting information related to the mathematical topics under discussion. Many of these are of a contemporary nature and, as such, they help students understand that math is an interesting and dynamic discipline that plays an important role in their daily lives.

TAKE NOTE

The alternative procedure for constructing a truth table, as described to the right, generally requires less writing, less time, and less effort than the truth table procedure that was used in Examples 1 and 2.

Take Note

These notes alert students to a point requiring special attention, or they are used to amplify the concepts currently being developed.

CALCULATOR NOTE

Some calculators display $\frac{7}{27}$ as 0.25925925926. However, the last digit 6 is not correct. It is a result of the rounding process. The actual decimal representation of $\frac{7}{27}$ is the decimal 0.259259... or $0.\overline{259}$, in which the digits continue to repeat the 259 pattern forever.

Calculator Note

These notes provide information about how to use the various features of a calculator.

Instructor Resources

Annotated Instructor's Edition (ISBN 978-1-305-96559-1): The Annotated Instructor's Edition features answers to all problems in the book.

Complete Solutions Manual: This manual contains complete solutions to all the problems in the text. Available on the Instructor Companion Site.

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Instructor Companion Site: This collection of book-specific lecture and class tools is available online at www.cengage.com/login. Access and download PowerPoint presentations, the solutions manual, and more.

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AIM for Success

Welcome to *Mathematical Excursions,* Fourth Edition. As you begin this course, we know two important facts: (1) You want to succeed. (2) We want you to succeed. In order to accomplish these goals, an effort is required from each of us. For the next few pages, we are going to show you what is required of you to achieve your goal and how we have designed this text to help you succeed.

Motivation

One of the most important keys to success is motivation. We can try to motivate you by offering interesting or important ways that you can benefit from mathematics. But, in the end, the motivation must come from you. On the first day of class it is easy to be motivated. Eight weeks into the term, it is harder to keep that motivation.

To stay motivated, there must be outcomes from this course that are worth your time, money, and energy. List some reasons you are taking this course. Do not make a mental list—actually write them out. Do this now.

Although we hope that one of the reasons you listed was an interest in mathematics, we know that many of you are taking this course because it is required to graduate, it is a prerequisite for a course you must take, or because it is required for your major. If you are motivated to graduate or complete the requirements for your major, then use that motivation to succeed in this course. Do not become distracted from your goal to complete your education!

TAKE NOTE

Motivation alone will not lead to success. For instance, suppose a person who cannot swim is placed in a boat, taken out to the middle of a lake, and then thrown overboard. That person has a lot of motivation to swim but there is a high likelihood the person will drown without some help. Motivation gives us the desire to learn but is not the same as learning.

Commitment

To be successful, you must make a commitment to succeed. This means devoting time to math so that you achieve a better understanding of the subject.

List some activities (sports, hobbies, talents such as dance, art, or music) that you enjoy and at which you would like to become better. Do this now.

Next to these activities, put the number of hours each week that you spend practicing these activities.

Whether you listed surfing or sailing, aerobics or restoring cars, or any other activity you enjoy, note how many hours a week you spend on each activity. To succeed in math, you must be willing to commit the same amount of time. Success requires some sacrifice.

The "I Can't Do Math" Syndrome

There may be things you cannot do, such as lift a two-ton boulder. You can, however, do math. It is much easier than lifting the two-ton boulder. When you first learned the activities you listed above, you probably could not do them well. With practice, you got better. With practice, you will be better at math. Stay focused, motivated, and committed to success.

It is difficult for us to emphasize how important it is to overcome the "I Can't Do Math Syndrome." If you listen to interviews of very successful athletes after a particularly bad performance, you will note that they focus on the positive aspect of what they did, not the negative. Sports psychologists encourage athletes to always be positive—to have a "can do" attitude. You need to develop this attitude toward math.

Strategies for Success

Know the Course Requirements To do your best in this course, you must know exactly what your instructor requires. Course requirements may be stated in a *syllabus*, which is a printed outline of the main topics of the course, or they may be presented orally. When they are listed in a syllabus or on other printed pages, keep them in a safe place. When they are presented orally, make sure to take complete notes. In either case, it is important that you understand them completely and follow them exactly. Be sure you know the answer to each of the following questions.

1. What is your instructor's name?
2. Where is your instructor's office?
3. At what times does your instructor hold office hours?
4. Besides the textbook, what other materials does your instructor require?
5. What is your instructor's attendance policy?
6. If you must be absent from a class meeting, what should you do before returning to class? What should you do when you return to class?
7. What is the instructor's policy regarding collection or grading of homework assignments?
8. What options are available if you are having difficulty with an assignment? Is there a math tutoring center?
9. If there is a math lab at your school, where is it located? What hours is it open?
10. What is the instructor's policy if you miss a quiz?
11. What is the instructor's policy if you miss an exam?
12. Where can you get help when studying for an exam?

Remember: Your instructor wants to see you succeed. If you need help, ask! Do not fall behind. If you were running a race and fell behind by 100 yards, you may be able to catch up, but it will require more effort than if you had not fallen behind.

Time Management We know that there are demands on your time. Family, work, friends, and entertainment all compete for your time. We do not want to see you receive poor job evaluations because you are studying math. However, it is also true that we do not want to see you receive poor math test scores because you devoted too much time to work. When several competing and important tasks require your time and energy, the only way to manage the stress of being successful at both is to manage your time efficiently.

Instructors often advise students to spend twice the amount of time outside of class studying as they spend in the classroom. Time management is important if you are to accomplish this goal and succeed in school. The following activity is intended to help you structure your time more efficiently.

Take out a sheet of paper and list the names of each course you are taking this term, the number of class hours each course meets, and the number of hours you should spend outside of class studying course materials. Now create a weekly calendar with the days of the week across the top and each hour of the day in a vertical column. Fill in the calendar with the hours you are in class, the hours you spend at work, and other commitments such as sports practice, music lessons, or committee meetings. Then fill in the hours that are more flexible, such as study time, recreation, and meal times.

TAKE NOTE

Besides time management, there must be realistic ideas of how much time is available. There are very few people who can *successfully* work full-time and go to school full-time. If you work 40 hours a week, take 15 units, spend the recommended study time given at the right, and sleep 8 hours a day, you use over 80% of the available hours in a week. That leaves less than 20% of the hours in a week for family, friends, eating, recreation, and other activities.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
10–11 A.M.	History	Rev Spanish	History	Rev Span Vocab	History	Jazz Band	
11–12 P.M.	Rev History	Spanish	Study group	Spanish	Math tutor	Jazz Band	
12–1 P.M.	Math		Math		Math		Soccer

We know that many of you must work. If that is the case, realize that working 10 hours a week at a part-time job is equivalent to taking a three-unit class. If you must work, consider letting your education progress at a slower rate to allow you to be successful at both work and school. There is no rule that says you must finish school in a certain time frame.

Schedule Study Time As we encouraged you to do by filling out the time management form, schedule a certain time to study. You should think of this time like being at work or class. Reasons for "missing study time" should be as compelling as reasons for missing work or class. "I just didn't feel like it" is not a good reason to miss your scheduled study time. Although this may seem like an obvious exercise, list a few reasons you might want to study. Do this now.

Of course we have no way of knowing the reasons you listed, but from our experience one reason given quite frequently is "To pass the course." There is nothing wrong with that reason. If that is the most important reason for you to study, then use it to stay focused.

One method of keeping to a study schedule is to form a ***study group***. Look for people who are committed to learning, who pay attention in class, and who are punctual. Ask them to join your group. Choose people with similar educational goals but different methods of learning. You can gain from seeing the material from a new perspective. Limit groups to four or five people; larger groups are unwieldy.

There are many ways to conduct a study group. Begin with the following suggestions and see what works best for your group.

1. Test each other by asking questions. Each group member might bring two or three sample test questions to each meeting.
2. Practice teaching each other. Many of us who are teachers learned a lot about our subject when we had to explain it to someone else.
3. Compare class notes. You might ask other students about material in your notes that is difficult for you to understand.
4. Brainstorm test questions.
5. Set an agenda for each meeting. Set approximate time limits for each agenda item and determine a quitting time.

And now, probably the most important aspect of studying is that it should be done in relatively small chunks. If you can study only three hours a week for this course (probably not enough for most people), do it in blocks of one hour on three separate days, preferably after class. Three hours of studying on a Sunday is not as productive as three hours of paced study.

Features of This Text That Promote Success

Preparing for Class Before the class meeting in which your professor begins a new chapter, you should read the title of each section. Next, browse through the chapter material, being sure to note each word in bold type. These words indicate important concepts that you must know to learn the material. Do not worry about trying to understand all the material. Your professor is there to assist you with that endeavor. The purpose of browsing through the material is so that your brain will be prepared to accept and organize the new information when it is presented to you.

Math Is Not a Spectator Sport To learn mathematics you must be an active participant. Listening and watching your professor do mathematics is not enough. Mathematics requires that you interact with the lesson you are studying. If you have been writing down the things we have asked you to do, you were being interactive. There are other ways this textbook has been designed so that you can be an active learner.

Check Your Progress One of the key instructional features of this text is a completely worked-out example followed by a *Check Your Progress*.

EXAMPLE 8 **Applications of the Blood Transfusion Table**

Use the blood transfusion table and Figures 2.3 and 2.4 to answer the following questions.

a. Can Sue safely be given a type O+ blood transfusion?

b. Why is a person with type O− blood called a *universal donor?*

Solution

a. Sue's blood type is A−. The blood transfusion table shows that she can safely receive blood only if it is type A− or type O−. Thus it is not safe for Sue to receive type O+ blood in a blood transfusion.

b. The blood transfusion table shows that all eight blood types can safely receive type O− blood. Thus a person with type O− blood is said to be a universal donor.

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Note that each Example is completely worked out and the *Check Your Progress* following the example is not. Study the worked-out example carefully by working through each step. You should do this with paper and pencil.

Now work the *Check Your Progress*. If you get stuck, refer to the page number following the word *Solution*, which directs you to the page on which the *Check Your Progress* is solved—a complete worked-out solution is provided. Try to use the given solution to get a hint for the step you are stuck on. Then try to complete your solution.

When you have completed the solution, check your work against the solution we provide.

CHECK YOUR PROGRESS 8 Use the blood transfusion table and Figures 2.3 and 2.4 to answer the following questions.

a. Is it safe for Alex to receive type A− blood in a blood transfusion?

b. What blood type do you have if you are classified as a *universal recipient?*

Solution See page S6.

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Be aware that frequently there is more than one way to solve a problem. Your answer, however, should be the same as the given answer. If you have any question as to whether your method will "always work," check with your instructor or with someone in the math center.

Remember: Be an active participant in your learning process. When you are sitting in class watching and listening to an explanation, you may think that you understand. However, until you actually try to do it, you will have no confirmation of the new knowledge or skill. Most of us have had the experience of sitting in class thinking we knew how to do something only to get home and realize we didn't.

Rule Boxes Pay special attention to definitions, theorems, formulas, and procedures that are presented in a rectangular box, because they generally contain the most important concepts in each section.

Simple Interest Formula

The simple interest formula is

$$I = Prt$$

where I is the interest, P is the principal, r is the interest rate, and t is the time period.

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Chapter Exercises When you have completed studying a section, do the section exercises. Math is a subject that needs to be learned in small sections and practiced continually in order to be mastered. Doing the exercises in each exercise set will help you master the problem-solving techniques necessary for success. As you work through the exercises, check your answers to the odd-numbered exercises against those in the back of the book.

Preparing for a Test There are important features of this text that can be used to prepare for a test.

- Chapter Summary
- Chapter Review Exercises
- Chapter Test

After completing a chapter, read the Chapter Summary. (See page 99 for the Chapter 2 Summary.) This summary highlights the important topics covered in each section of the chapter. Each concept is paired with page numbers of examples that illustrate the concept and exercises that will provide you with practice on the skill or technique.

Following the Chapter Summary are Chapter Review Exercises (see page 101). Doing the review exercises is an important way of testing your understanding of the chapter. The answer to each review exercise is given at the back of the book, along with, in brackets, the section reference from which the question was taken (see page A5). After checking your answers, restudy any section from which a question you missed was taken. It may be helpful to retry some of the exercises for that section to reinforce your problem-solving techniques.

Each chapter ends with a Chapter Test (see page 103). This test should be used to prepare for an exam. We suggest that you try the Chapter Test a few days before your actual exam. Take the test in a quiet place and try to complete the test in the same amount of time you will be allowed for your exam. When taking the Chapter Test, practice the strategies of successful test takers: (1) scan the entire test to get a feel for the questions; (2) read the directions carefully; (3) work the problems that are easiest for you first; and perhaps most importantly, (4) try to stay calm.

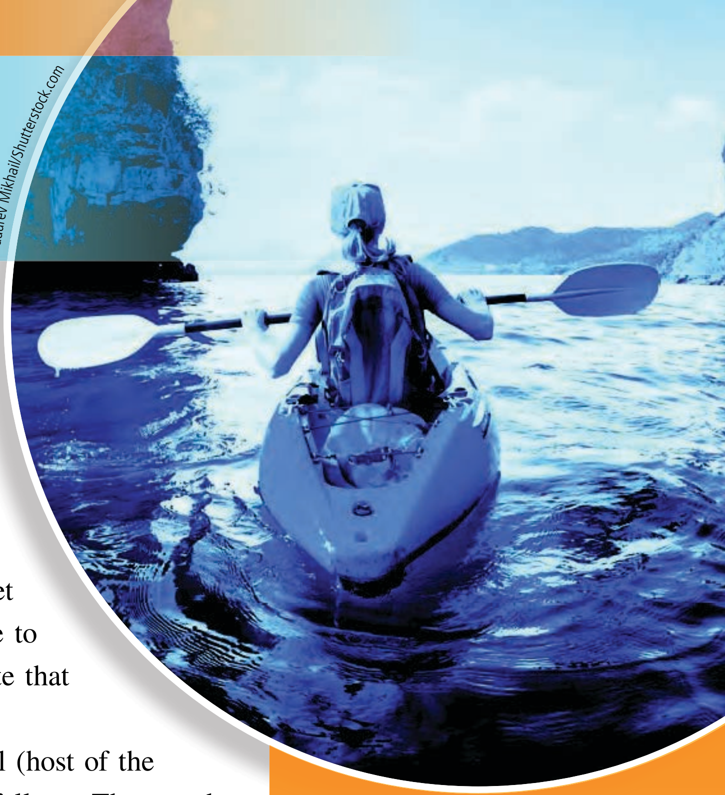
When you have completed the Chapter Test, check your answers for each exercise (see page A6). Next to each answer is, in brackets, the reference to the section from which the question was taken and an example reference for each exercise. The section references indicate the section or sections where you can locate the concepts needed to solve a given exercise, and the example reference allows you to easily find an example that is similar to the given test exercise. If you missed a question, review the material in that section and rework some of the exercises from that section. This will strengthen your ability to perform the skills in that section.

Your career goal goes here. →

Is it difficult to be successful? YES! Successful music groups, artists, professional athletes, teachers, sociologists, chefs, and ___________ have to work very hard to achieve their goals. They focus on their goals and ignore distractions. The things we ask you to do to achieve success take time and commitment. We are confident that if you follow our suggestions, you will succeed.

1

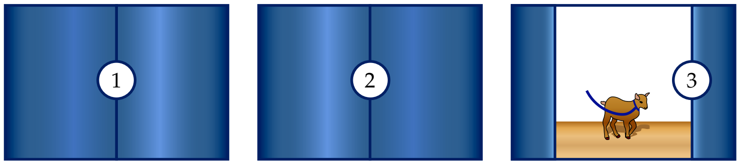
Problem Solving



Dudarev Mikhail/Shutterstock.com

Most occupations require good problem-solving skills. For instance, architects and engineers must solve many complicated problems as they design and construct modern buildings that are aesthetically pleasing, functional, and that meet stringent safety requirements. Two goals of this chapter are to help you become a better problem solver and to demonstrate that problem solving can be an enjoyable experience.

One problem that many have enjoyed is the Monty Hall (host of the game show *Let's Make a Deal*) problem, which is stated as follows. The grand prize in *Let's Make a Deal* is behind one of three doors. Less desirable prizes (for instance, a goat and a box of candy) are behind the other two doors. You select one of the doors, say door 1. Monty Hall reveals one of the less desirable prizes behind one of the other doors. You are then given the opportunity either to stay with your original choice or to choose the remaining closed door.



Example: You choose door 1. Monty Hall reveals a goat behind door 3. You can stay with door 1 or switch to door 2.

Marilyn vos Savant, author of the "Ask Marilyn" column featured in *Parade Magazine,* analyzed this problem,[1] claiming that you *double* your chances of winning the grand prize by switching to the other closed door. Many readers, including some mathematicians, responded with arguments that contradicted Marilyn's analysis.

What do you think? Do you have a better chance of winning the grand prize by switching to the other closed door or staying with your original choice?

Of course there is also the possibility that it does not matter, if the chances of winning are the same with either strategy.

Discuss the Monty Hall problem with some of your friends and classmates. Is everyone in agreement? Additional information on this problem is given in Exploration Exercise 54 on page 14.

Photo by Timothy White

Marilyn vos Savant

[1] "Ask Marilyn," *Parade Magazine*, September 9, 1990, p. 15.

SECTION 1.1 Inductive and Deductive Reasoning

Inductive Reasoning

The type of reasoning that forms a conclusion based on the examination of specific examples is called *inductive reasoning*. The conclusion formed by using inductive reasoning is a **conjecture**, since it may or may not be correct.

> **Inductive Reasoning**
>
> **Inductive reasoning** is the process of reaching a general conclusion by examining specific examples.

When you examine a list of numbers and predict the next number in the list according to some pattern you have observed, you are using inductive reasoning.

EXAMPLE 1 Use Inductive Reasoning to Predict a Number

Use inductive reasoning to predict the next number in each of the following lists.

a. 3, 6, 9, 12, 15, ? **b.** 1, 3, 6, 10, 15, ?

Solution

a. Each successive number is 3 larger than the preceding number. Thus we predict that the next number in the list is 3 larger than 15, which is 18.

b. The first two numbers differ by 2. The second and the third numbers differ by 3. It appears that the difference between any two numbers is always 1 more than the preceding difference. Since 10 and 15 differ by 5, we predict that the next number in the list will be 6 larger than 15, which is 21.

CHECK YOUR PROGRESS 1 Use inductive reasoning to predict the next number in each of the following lists.

a. 5, 10, 15, 20, 25, ? **b.** 2, 5, 10, 17, 26, ?

Solution See page S1.

Inductive reasoning is not used just to predict the next number in a list. In Example 2 we use inductive reasoning to make a conjecture about an arithmetic procedure.

EXAMPLE 2 Use Inductive Reasoning to Make a Conjecture

Consider the following procedure: Pick a number. Multiply the number by 8, add 6 to the product, divide the sum by 2, and subtract 3.

Complete the above procedure for several different numbers. Use inductive reasoning to make a conjecture about the relationship between the size of the resulting number and the size of the original number.

Solution

Suppose we pick 5 as our original number. Then the procedure would produce the following results:

Original number:	5
Multiply by 8:	$8 \times 5 = 40$
Add 6:	$40 + 6 = 46$
Divide by 2:	$46 \div 2 = 23$
Subtract 3:	$23 - 3 = 20$

TAKE NOTE

In Example 5, we will use a deductive method to verify that the procedure in Example 2 always yields a result that is four times the original number.

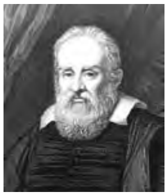
We started with 5 and followed the procedure to produce 20. Starting with 6 as our original number produces a final result of 24. Starting with 10 produces a final result of 40. Starting with 100 produces a final result of 400. In each of these cases the resulting number is four times the original number. We *conjecture* that following the given procedure produces a number that is four times the original number.

CHECK YOUR PROGRESS 2 Consider the following procedure: Pick a number. Multiply the number by 9, add 15 to the product, divide the sum by 3, and subtract 5.

Complete the above procedure for several different numbers. Use inductive reasoning to make a conjecture about the relationship between the size of the resulting number and the size of the original number.

Solution See page S1.

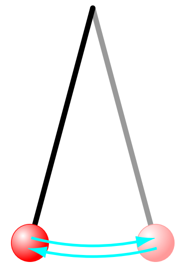
HISTORICAL NOTE


Hulton Archive/Getty Images

Galileo Galilei (găl′-ə-lā′ē′) entered the University of Pisa to study medicine at the age of 17, but he soon realized that he was more interested in the study of astronomy and the physical sciences. Galileo's study of pendulums assisted in the development of pendulum clocks.

Scientists often use inductive reasoning. For instance, Galileo Galilei (1564–1642) used inductive reasoning to discover that the time required for a pendulum to complete one swing, called the *period* of the pendulum, depends on the length of the pendulum. Galileo did not have a clock, so he measured the periods of pendulums in "heartbeats." The following table shows some results obtained for pendulums of various lengths. For the sake of convenience, a length of 10 inches has been designated as 1 unit.

Length of pendulum, in units	Period of pendulum, in heartbeats
1	1
4	2
9	3
16	4
25	5
36	6



The period of a pendulum is the time it takes for the pendulum to swing from left to right and back to its original position.

EXAMPLE 3 Use Inductive Reasoning to Solve an Application

Use the data in the above table and inductive reasoning to answer each of the following questions.

a. If a pendulum has a length of 49 units, what is its period?

b. If the length of a pendulum is quadrupled, what happens to its period?

Solution

a. In the table, each pendulum has a period that is the square root of its length. Thus we conjecture that a pendulum with a length of 49 units will have a period of 7 heartbeats.

b. In the table, a pendulum with a length of 4 units has a period that is twice that of a pendulum with a length of 1 unit. A pendulum with a length of 16 units has a period that is twice that of a pendulum with a length of 4 units. It appears that quadrupling the length of a pendulum doubles its period.

Velocity of tsunami, in feet per second	Height of tsunami, in feet
6	4
9	9
12	16
15	25
18	36
21	49
24	64

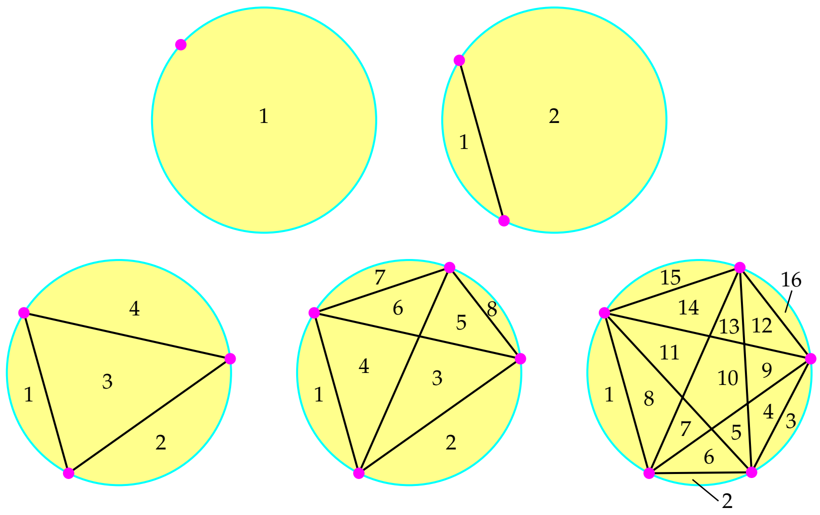
CHECK YOUR PROGRESS 3 A tsunami is a sea wave produced by an underwater earthquake. The height of a tsunami as it approaches land depends on the velocity of the tsunami. Use the table at the left and inductive reasoning to answer each of the following questions.

a. What happens to the height of a tsunami when its velocity is doubled?

b. What should be the height of a tsunami if its velocity is 30 feet per second?

Solution See page S1.

Conclusions based on inductive reasoning may be incorrect. As an illustration, consider the circles shown below. For each circle, all possible line segments have been drawn to connect each dot on the circle with all the other dots on the circle.



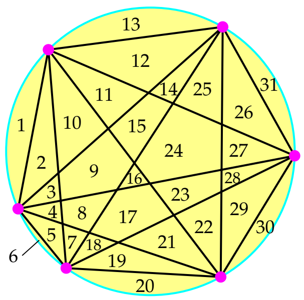
The maximum numbers of regions formed by connecting dots on a circle

TAKE NOTE

To produce the maximum number of regions, the dots on a circle must be placed so that no three line segments that connect the dots intersect at a single point.

For each circle, count the number of regions formed by the line segments that connect the dots on the circle. Your results should agree with the results in the following table.

Number of dots	1	2	3	4	5	6
Maximum number of regions	1	2	4	8	16	?



The line segments connecting six dots on a circle yield a maximum of 31 regions.

There appears to be a pattern. Each additional dot seems to double the number of regions. Guess the maximum number of regions you expect for a circle with six dots. Check your guess by counting the maximum number of regions formed by the line segments that connect six dots on a *large* circle. Your drawing will show that for six dots, the maximum number of regions is 31 (see the figure at the left), not 32 as you may have guessed. With seven dots the maximum number of regions is 57. This is a good example to keep in mind. Just because a pattern holds true for a few cases, it does not mean the pattern will continue. When you use inductive reasoning, you have no guarantee that your conclusion is correct.

Counterexamples

A statement is a true statement provided that it is true in all cases. If you can find *one case* for which a statement is not true, called a **counterexample**, then the statement is a false statement. In Example 4 we verify that each statement is a false statement by finding a counterexample for each.

EXAMPLE 4 Find a Counterexample

Verify that each of the following statements is a false statement by finding a counterexample.

For all numbers x:

a. $|x| > 0$ **b.** $x^2 > x$ **c.** $\sqrt{x^2} = x$

Solution

A statement may have many counterexamples, but we need only find one counterexample to verify that the statement is false.

a. Let $x = 0$. Then $|0| = 0$. Because 0 is not greater than 0, we have found a counterexample. Thus "for all numbers x, $|x| > 0$" is a false statement.

b. For $x = 1$ we have $1^2 = 1$. Since 1 is not greater than 1, we have found a counterexample. Thus "for all numbers x, $x^2 > x$" is a false statement.

c. Consider $x = -3$. Then $\sqrt{(-3)^2} = \sqrt{9} = 3$. Since 3 is not equal to -3, we have found a counterexample. Thus "for all numbers x, $\sqrt{x^2} = x$" is a false statement.

CHECK YOUR PROGRESS 4 Verify that each of the following statements is a false statement by finding a counterexample for each.

For all numbers x:

a. $\frac{x}{x} = 1$ **b.** $\frac{x+3}{3} = x + 1$ **c.** $\sqrt{x^2 + 16} = x + 4$

Solution See page S1.

QUESTION How many counterexamples are needed to prove that a statement is false?

Deductive Reasoning

Another type of reasoning is called *deductive reasoning.* Deductive reasoning is distinguished from inductive reasoning in that it is the process of reaching a conclusion by applying general principles and procedures.

Deductive Reasoning

Deductive reasoning is the process of reaching a conclusion by applying general assumptions, procedures, or principles.

TAKE NOTE

Example 5 is the same as Example 2, on page 2, except in Example 5 we use deductive reasoning, instead of inductive reasoning.

EXAMPLE 5 Use Deductive Reasoning to Establish a Conjecture

Use deductive reasoning to show that the following procedure produces a number that is four times the original number.

Procedure: Pick a number. Multiply the number by 8, add 6 to the product, divide the sum by 2, and subtract 3.

Solution

Let n represent the original number.

Multiply the number by 8:	$8n$
Add 6 to the product:	$8n + 6$
Divide the sum by 2:	$\frac{8n+6}{2} = 4n + 3$
Subtract 3:	$4n + 3 - 3 = 4n$

We started with n and ended with $4n$. The procedure given in this example produces a number that is four times the original number.

ANSWER One

CHECK YOUR PROGRESS 5 Use deductive reasoning to show that the following procedure produces a number that is three times the original number.

Procedure: Pick a number. Multiply the number by 6, add 10 to the product, divide the sum by 2, and subtract 5. *Hint:* Let n represent the original number.

Solution See page S1.

MATH MATTERS Deductive Reasoning in Mathematics

You may have observed that some of your math classes made extensive use of deductive reasoning to prove theorems and solve problems. The following quote by the mathematician Paul R. Halmos (1916–2006) advocates that you not limit yourself to only using deductive reasoning to prove theorems.

"Mathematics is not a deductive science—that's a cliché. When you try to prove a theorem, you don't just list the hypotheses, and then start to reason. What you do is trial and error, experimentation, guesswork."

I Want to be a Mathematician: An Automathography (1985).

Inductive Reasoning vs. Deductive Reasoning

In Example 6 we analyze arguments to determine whether they use inductive or deductive reasoning.

EXAMPLE 6 Determine Types of Reasoning

Determine whether each of the following arguments is an example of inductive reasoning or deductive reasoning.

a. During the past 10 years, a tree has produced plums every other year. Last year the tree did not produce plums, so this year the tree will produce plums.

b. All home improvements cost more than the estimate. The contractor estimated that my home improvement will cost $35,000. Thus my home improvement will cost more than $35,000.

Solution

a. This argument reaches a conclusion based on specific examples, so it is an example of inductive reasoning.

b. Because the conclusion is a specific case of a general assumption, this argument is an example of deductive reasoning.

CHECK YOUR PROGRESS 6 Determine whether each of the following arguments is an example of inductive reasoning or deductive reasoning.

a. All Gillian Flynn novels are worth reading. The novel *Gone Girl* is a Gillian Flynn novel. Thus *Gone Girl* is worth reading.

b. I know I will win a jackpot on this slot machine in the next 10 tries, because it has not paid out any money during the last 45 tries.

Solution See page S1.

Logic Puzzles

Logic puzzles, similar to the one in Example 7, can be solved by using deductive reasoning and a chart that enables us to display the given information in a visual manner.

EXAMPLE 7 Solve a Logic Puzzle

Each of four neighbors, Sean, Maria, Sarah, and Brian, has a different occupation (editor, banker, chef, or dentist). From the following clues, determine the occupation of each neighbor.

1. Maria gets home from work after the banker but before the dentist.
2. Sarah, who is the last to get home from work, is not the editor.
3. The dentist and Sarah leave for work at the same time.
4. The banker lives next door to Brian.

Solution

From clue 1, Maria is not the banker or the dentist. In the following chart, write X1 (which stands for "ruled out by clue 1") in the Banker and the Dentist columns of Maria's row.

	Editor	Banker	Chef	Dentist
Sean				
Maria		X1		X1
Sarah				
Brian				

From clue 2, Sarah is not the editor. Write X2 (ruled out by clue 2) in the Editor column of Sarah's row. We know from clue 1 that the banker is not the last to get home, and we know from clue 2 that Sarah is the last to get home; therefore, Sarah is not the banker. Write X2 in the Banker column of Sarah's row.

	Editor	Banker	Chef	Dentist
Sean				
Maria		X1		X1
Sarah	X2	X2		
Brian				

From clue 3, Sarah is not the dentist. Write X3 for this condition. There are now Xs for three of the four occupations in Sarah's row; therefore, Sarah must be the chef. Place a ✓ in that box. Since Sarah is the chef, none of the other three people can be the chef. Write X3 for these conditions. There are now Xs for three of the four occupations in Maria's row; therefore, Maria must be the editor. Insert a ✓ to indicate that Maria is the editor, and write X3 twice to indicate that neither Sean nor Brian is the editor.

	Editor	Banker	Chef	Dentist
Sean	X3		X3	
Maria	✓	X1	X3	X1
Sarah	X2	X2	✓	X3
Brian	X3		X3	

From clue 4, Brian is not the banker. Write X4 for this condition. See the following table. Since there are three Xs in the Banker column, Sean must be the banker. Place a

✓ in that box. Thus Sean cannot be the dentist. Write X4 in that box. Since there are 3 Xs in the Dentist column, Brian must be the dentist. Place a ✓ in that box.

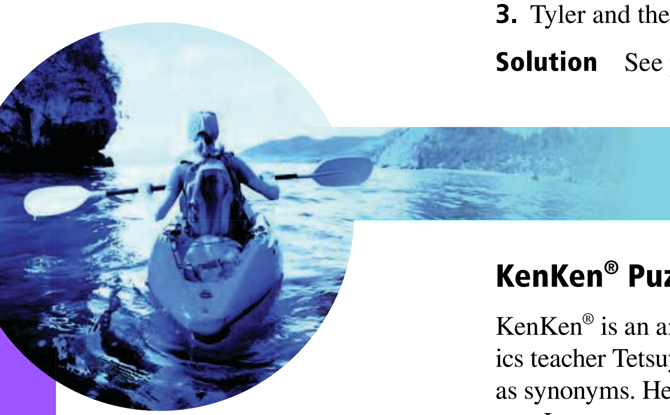
	Editor	Banker	Chef	Dentist
Sean	X3	✓	X3	X4
Maria	✓	X1	X3	X1
Sarah	X2	X2	✓	X3
Brian	X3	X4	X3	✓

Sean is the banker, Maria is the editor, Sarah is the chef, and Brian is the dentist.

CHECK YOUR PROGRESS 7 Brianna, Ryan, Tyler, and Ashley were recently elected as the new class officers (president, vice president, secretary, treasurer) of the sophomore class at Summit College. From the following clues, determine which position each holds.

1. Ashley is younger than the president but older than the treasurer.
2. Brianna and the secretary are both the same age, and they are the youngest members of the group.
3. Tyler and the secretary are next-door neighbors.

Solution See page S1.



EXCURSION

KenKen® Puzzles: An Introduction

KenKen® is an arithmetic-based logic puzzle that was invented by the Japanese mathematics teacher Tetsuya Miyamoto in 2004. The noun “ken” has “knowledge” and “awareness” as synonyms. Hence, KenKen translates as knowledge squared, or awareness squared.

In recent years the popularity of KenKen has increased at a dramatic rate. More than a million KenKen puzzle books have been sold, and KenKen puzzles now appear in many popular newspapers, including the *New York Times* and the *Boston Globe*.

KenKen puzzles are similar to Sudoku puzzles, but they also require you to perform arithmetic to solve the puzzle.

Rules for Solving a KenKen Puzzle

For a 3 by 3 puzzle, fill in each box (square) of the grid with one of the numbers 1, 2, or 3.

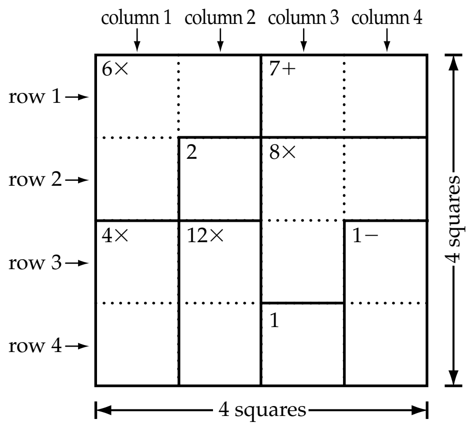
For a 4 by 4 puzzle, fill in each square of the grid with one of the numbers 1, 2, 3, or 4.

For a n by n puzzle, fill in each square of the grid with one of the numbers 1, 2, 3, ..., n.

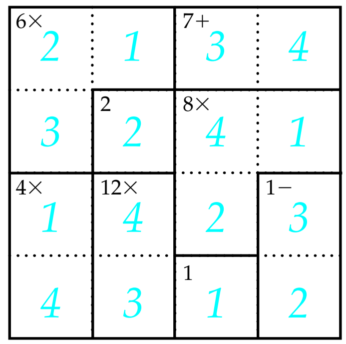
Grids range in size from a 3 by 3 up to a 9 by 9.

- Do not repeat a number in any row or column.
- The numbers in each heavily outlined set of squares, called **cages**, must combine (in some order) to produce the **target number** in the top left corner of the cage using the mathematical operation indicated.
- Cages with just one square should be filled in with the target number.
- A number can be repeated within a cage as long as it is not in the same row or column.

Here is a 4 by 4 puzzle and its solution. Properly constructed puzzles have a unique solution.



A 4 by 4 puzzle with 8 cages



The solution to the puzzle

Basic Puzzle Solution Strategies

Single-Square Cages Fill cages that consist of a single square with the target number for that square.

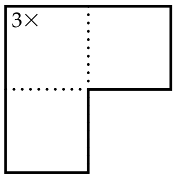
Cages with Two Squares Next examine the cages with exactly two squares. Many cages that cover two squares will only have two digits that can be used to fill the cage. For instance, in a 5 by 5 puzzle, a **20×** cage with exactly two squares can only be filled with 4 and 5 or 5 and 4.

Large or Small Target Numbers Search for cages that have an unusually large or small target number. These cages generally have only a few combinations of numbers that can be used to fill the cage.

Examples:

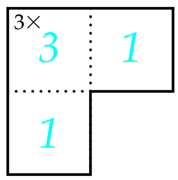
In a 6 by 6 puzzle, a **120×** cage with exactly three squares can only be filled with 4, 5, and 6.

A **3+** cage with exactly two squares can only be filled with 1 and 2.



An L-shaped cage

Duplicate Digit in a Cage Consider the **3×** cage shown at the left. The digits 1, 1, and 3 produce a product of 3; however, we cannot place the two 1s in the same row or the same column. Thus the only way to fill the squares is to place the 3 in the corner of the L-shaped cage as shown below. *Remember:* A digit can occur more than once in a cage, provided that it does not appear in the same row or in the same column.



Remember the Following Rules

In an *n* by *n* puzzle, each row and column must contain every digit from 1 to *n*.

In a two-square cage that involves subtraction or division, the order of the numbers in the cage is not important. For instance, a **3–** cage with two squares could be filled with 4 and 1 or with 1 and 4. A **3÷** cage with two squares could be filled with 3 and 1 or with 1 and 3.